# The ratio $F_2^n/F_2^p$ from the analysis of data using a new scaling variable

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#### Abstract

We analyze the proton and deuteron structure functions at large x using a recently introduced scaling variable  $\bar{x}$ . This variable includes power corrections to x-scaling, and thus allows us to reach the Bjorken limit at moderate  $Q^2$ . Using available data we extract the ratio  $F_2^n(x)/F_2^p(x)$  for  $x \leq 0.85$ . Contrary to earlier expectations this ratio tends to the value  $\sim 2/3$  for  $x \to 1$ , which corresponds to the quark model prediction for equal distributions of valence quarks.

According to the quark-parton model, the structure functions of hadrons in the Bjorken limit  $(Q^2 = \mathbf{q}^2 - \nu^2 \to \infty \text{ and } x = Q^2/2M\nu = \text{const})$  are directly related to the parton distributions  $q_i(x)$ . For instance

$$F_2(x, Q^2) \to F_2(x) = \sum_i e_i^2 x q_i(x),$$
 (1)

where the sum is over the partons, whose charges are  $e_i$ . In the region  $x \to 1$ , the contribution of sea quarks can be neglected. Then assuming the same distribution of the valence quarks, one easily finds from Eq. (1) that the neutron-to-proton ratio  $F_2^n(x)/F_2^p(x)$  approaches 2/3 for  $x \to 1$ . If, however, the quark distributions are different, one can establish only upper and lower limits for this ratio,  $1/4 < F_2^n/F_2^p < 4$ , which follow from isospin invariance

[1]. The existing data show considerable  $Q^2$ -dependence of the structure functions which is attributed mainly to the QCD logarithmic corrections to Bjorken scaling. However, at high-x the scaling violations are dominated by power corrections  $\propto 1/Q^2$  (higher twist and target mass effects), which are difficult to evaluate. Therefore, in order to check the parton model predictions, one needs to obtain the structure functions at high  $Q^2$ , where these corrections are small. At present, high  $Q^2$  structure functions  $(Q^2 \simeq 250 \text{ (GeV/c})^2)$  extracted from BCDMS [2] and NMC [3] data are available only for  $x \lesssim 0.7$ . The ratio  $F_2^n(x)/F_2^p(x)$  obtained from an analysis of these data [4,5] shows steady decrease with x. Thus, it is usually assumed that this ratio would reach its lower bound,  $F_2^n/F_2^p \to 1/4$ , for  $x \to 1$ . This corresponds to  $d(x)/u(x) \to 0$  for  $x \to 1$ , where d(x) and u(x) are the distribution functions for up and down quarks in the proton. To check this assumption one needs data for the structure functions at  $x \gtrsim 0.7$ . The latter are now available only at moderate values of momentum transfer,  $Q^2 \lesssim 30 \text{ (GeV/c})^2$  [6–9], and exhibit very strong  $Q^2$ -dependence.

A part of the  $Q^2$ -dependence of the structure functions, generated by the target mass corrections, is usually accounted for by using the Nachtmann scaling variable [10,11]

$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}},\tag{2}$$

instead of the Bjorken variable x. The question is whether the target mass effect is responsible for a major part of the scaling violation at high x. If so, the replacement of x by  $\xi$  in structure functions would allow us to reach the scaling limit already at moderate  $Q^2$ . However, the analysis of recent high x SLAC data [9] in terms of the Nachtmann variable still reveals strong  $Q^2$ -dependence of the structure functions.

Besides the target mass effects, one can expect important nonperturbative effects from the confining interaction of the partons in the final state. Indeed, the partons are never free, so that the system possesses a discrete spectrum in the final state. Although in the Bjorken limit the struck quark can be considered a free particle, the discreteness of the spectrum manifests itself in  $1/Q^2$  corrections to asymptotic structure functions [12,13]. One can expect that these corrections are significant in particular at high x, where lower-lying excitations should play an important role.

We have found [14] that the target mass and confining interaction effects in the final state can be effectively accounted for by taken the struck quark to be off-shell, with the same virtual mass before and after the virtual photon absorption. As a result, the Bjorken scaling variable x is replaced by a new scaling variable  $\bar{x} \equiv \bar{x}(x, Q^2)$ , which is the light-cone fraction of the off-shell struck quark. Explicitly,

$$\bar{x} = \frac{x + \sqrt{1 + 4M^2x^2/Q^2} - \sqrt{(1 - x)^2 + 4m_s^2x^2/Q^2}}{1 + \sqrt{1 + 4M^2x^2/Q^2}},$$
(3)

where M is the target mass and  $m_s$  is the invariant mass of spectator partons (quarks and gluons). For  $Q^2 \to \infty$  or for  $x \to 0$  the variable  $\bar{x}$  coincides with the Nachtmann variable  $\xi$ , Eq. (2). However, at finite  $Q^2$  these variables are quite different.

It follows from Eq. (3) that  $\bar{x}$  depends on the invariant spectator mass,  $m_s$ . The latter can be considered a function of the external parameters only [14]. In the limit  $x \to 1$  (elastic scattering) no gluons are emitted, and thus  $m_s \to m_0$ , the mass of a two-quark system (diquark). When x < 1, the spectator mass  $m_s$  increases due to gluon emission. For x close to 1,  $m_s$  can be approximated as

$$m_s^2 \simeq m_0^2 + C(1-x),$$
 (4)

where the coefficient  $C \sim (\text{GeV})^2$ . In the following we regard it as a phenomenological parameter, determined from the data.

Consider the proton and deuteron structure functions (per nucleon)  $F_2^{p,d}(x,Q^2)$  at large x. These are shown in Fig. 1a,b as functions of  $\bar{x}$  for  $Q^2=230$  (GeV/c)<sup>2</sup>, which is the maximal value of  $Q^2$  in the BCDMS measurements [2]. (Notice that for such high values of  $Q^2$  the variable  $\bar{x}$  is close to x, Eq. (3)). The data are taken from [2], where the solid lines correspond to a 15 parameter fit to BCDMS and NMC data [3]. For smaller values of  $Q^2$ , the violations of Bjorken scaling are very significant, Fig. 1c,d. By assuming that the major part of x-scaling violations at high x are correctly accounted for by the variable  $\bar{x}$ , the  $Q^2$ -dependence of the structure functions in this region is given by

$$F_2^{p,d}(x,Q^2) = F_2^{p,d}(\bar{x}(x,Q^2)). \tag{5}$$

Let us compare this result with two BCDMS and SLAC data bins for x=0.65, 0.75 [2,6], Fig. 1c,d, which are the largest values of x available in the BCDMS experiment. We find that these data are perfectly reproduced (the dashed lines in Fig. 1c,d), by taking the spectator mass  $m_s^2=0.75$  (GeV)<sup>2</sup> for x=0.75 and  $m_s^2=1.05$  (GeV)<sup>2</sup> for x=0.65. It is quite remarkable that the same values of  $m_s$  are obtained for proton and deuteron targets, although the corresponding structure functions are rather different. Using Eq. (4) one finds that these values of  $m_s$  determine the parameters  $m_0$  and C, namely C=3 (GeV)<sup>2</sup> and  $m_0=0$ . The latter implies that the spectator quarks are massless and collinear.

Using these values of  $m_0$  and C for definition of  $m_s$  in Eq. (3) we can study the structure functions for x > 0.75. Consider first the data for the proton structure function from the SLAC experiments [6–9] in the region x > 0.6 for  $5 \lesssim Q^2 \lesssim 30$  (GeV/c)<sup>2</sup>, plotted as a function of  $\xi$  and  $\bar{x}$  respectively, Fig. 2a,b. The data points close to the region of resonances were excluded by a requirement on the invariant mass of the final state, namely  $(M+\nu)^2 - \mathbf{q}^2 > (M+\Delta)^2$ , where  $\Delta = 300$  MeV. The solid line and the three data points in Fig. 2a,b correspond to the asymptotic structure function at  $Q^2 = 230$  (GeV/c)<sup>2</sup>, the same as in Fig. 1a. The analysis in terms of the Nachtmann scaling variable  $\xi$ , Fig. 2a, shows poor scaling. Moreover, the data points are far off the asymptotic structure function (the solid line). In contrast, the same data plotted as a function of  $\bar{x}$ , Fig. 2b, show excellent scaling. Also the data points completely coincide with the structure function at  $Q^2 = 230$  (GeV/c)<sup>2</sup>, available for x < 0.75. This agreement provides strong evidence that the  $\bar{x}$ -scaling is not accidental. We therefore propose that the data points in Fig. 2b represent a measurement of the asymptotic structure function for x > 0.75 as well.

Next, consider the deuteron structure function from the SLAC data [6–8], Fig. 3a,b. As in the previous case, we exclude the region of resonances by taking the invariant mass in the final state greater then  $M + \Delta$ , where  $\Delta$ =300 MeV. In addition, in order to avoid complications from binding and Fermi motion effects, we exclude from our analysis the data

points with x > 0.9. (This restriction is relevant only for the data [8]). Indeed, recent calculations of Melnitchouk et al. [15] show that the ratio  $2F_2^d/(F_2^p + F_2^n)$  is about 1.13 for x = 0.9 and  $Q^2 = 5$  (GeV/c)<sup>2</sup>, and it rapidly increases for x > 0.9. However, for x < 0.85, this ratio is within 5% of unity [16]. Fig. 3a shows the deuteron data plotted as a function of the Nachtmann variable  $\xi$ . The solid line and three data points show the structure function at  $Q^2 = 230$  (GeV/c)<sup>2</sup>, the same as in Fig. 1b. One finds that the data display no scaling and they are far from the solid line. The same data as a function of  $\bar{x}$  are shown in Fig. 3b. As in the proton case the data show very good scaling and do coincide with the structure function at  $Q^2 = 230$  (GeV/c)<sup>2</sup>. Unfortunately, there are no deuteron data in the high-x region for large  $Q^2$ , as for instance the proton data [9]. As a result, the available deuteron data allow us to determine the asymptotic structure function only up to x = 0.85.

Now with the asymptotic structure functions  $F_2(\bar{x}) = F_2(x) \equiv F_2(x, Q^2 \to \infty)$  found above, we can obtain the ratio  $F_2^n(x)/F_2^p(x)$ . For this purpose we parametrize the asymptotic structure functions as  $F_2^{p,d}(x) = \exp(-\sum_{i=0}^4 a_i x^i)$  and determine the parameters  $a_i$  from the best fit to the data in Figs. 2b, 3b. The resulting  $F_2^n/F_2^p$  ratio is shown in Fig. 4 by the solid line. The dotted lines are the error bars on the fit, which combine statistical and systematic uncertainties. The dashed line corresponds to  $F_2^n/F_2^p = 2/3$ . For a comparison, we show by the dot-dashed line a polynomial extrapolation of this ratio to large x, obtained from BCDMS and NMC data by assuming that  $F_2^n/F_2^p \to 1/4$  for  $x \to 1$  [4].

Our results shown in Fig. 4 demonstrate that contrary to earlier expectations, the ratio  $F_2^n/F_2^p$  does not approach its lower bound, but increases up to approximately 2/3. The latter is the quark model prediction, assuming identical distributions for each of the valence quarks. The accuracy of our results will be checked in future experiments, which will provide high  $Q^2$  data for the structure functions at large x.

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## REFERENCES

- [1] O. Nachtmann, Nucl. Phys. B38, 397 (1972).
- [2] BCDMS Collab., A.C. Benvenuti et al., Phys. Lett. B223, 485 (1989); Phys. Lett. B237, 592 (1989).
- [3] NMC Collab., P. Amaudruz et al., Phys. Lett. B295, 159 (1992).
- [4] BCDMS Collab., A.C. Benvenuti et al., Phys. Lett. B237, 599 (1989).
- [5] NMC Collab., P. Amaudruz et al., Phys. Lett. B371, 3 (1992).
- [6] L.W. Whitlow, Ph.D. Thesis, Stanford University, 1990, SLAC-REPORT-357 (1990).
- [7] L.W. Whitlow et al., Phys. Lett. B282, 475 (1992).
- [8] S.E. Rock et al., Phys. Rev. D46, 24 (1992).
- [9] P.E. Bosted *et al.*, Phys. Rev. D49, 3091 (1994).
- [10] O. Nachtmann, Nucl. Phys. B63, 237 (1973); B78, 455 (1974).
- [11] A. De Rujula, H. Georgi, and H.D. Politzer, Ann. Phys. (N.Y.) **109**, 315 (1977).
- [12] O.W. Greenberg, Phys. Rev. D47, 331 (1993).
- [13] S.A. Gurvitz and A.S. Rinat, Phys. Rev. C47, 2901 (1993).
- 14 S.A. Gurvitz, Phys. Rev. D, in press.
- [15] W. Melnitchouk, A.W. Schreiber and A.W. Thomas, Phys. Lett. B335, 11 (1994).
- [16] Similar small binding and Fermi motion effects in the deuteron structure function were also found in a recent phenomenological analysis of J. Gomez et al., Phys. Rev. D49, 4348 (1994).

#### **FIGURES**

- FIG. 1. (a,b)  $F_2^{p,d}(\bar{x}) = F_2^{p,d}(\bar{x},Q^2)$  at  $Q^2=230$  (GeV/c)<sup>2</sup>. The data are from BCDMS measurements [2]. The solid line is the 15 parameter fit [3] to BCDMS, NMC and SLAC data. (c,d) Proton and deuteron structure functions at constant x. The dashed lines show  $Q^2$ -dependence of the structure functions given by Eq. (5). The data are from BCDMS [2] and SLAC [6,7] experiments. The error bars show combined statistical and systematic errors.
- FIG. 2. (a) SLAC data for the proton structure function for  $5 \lesssim Q^2 \lesssim 30 \; (\text{GeV/c})^2$ , plotted as a function of the Nachtmann variable  $\xi$ , Eq. (2). The data with largest value of  $\xi$  are taken from recent measurements [9]. Three high-statistics data sets for  $Q^2=5.9$ , 7.9, and 9.8  $(\text{GeV/c})^2$ , taken from [8,9], are marked by "+", "x", and "#" respectively. The other data points are from [6,7]. The error bars show combined statistical and systematic uncertainties. Three data points marked by "o" and the solid curve are the same as in Fig. 1a, and show the structure function at  $Q^2=230 \; (\text{GeV/c})^2$ . (b). The same data plotted as a function of the scaling variable  $\bar{x}$ . The data points for  $5 \lesssim Q^2 \lesssim 30 \; (\text{GeV/c})^2$  coincide well with the structure function at  $Q^2=230 \; (\text{GeV/c})^2$ .
- FIG. 3. (a) SLAC data for deuteron structure function for  $5 \lesssim Q^2 \lesssim 30 \; (\text{GeV/c})^2$  plotted as a function of the Nachtmann variable  $\xi$ , Eq. (2). Four high-statistics data sets for  $Q^2=3.9, 5.9, 7.9,$  and  $9.8 \; (\text{GeV/c})^2$ , taken from [8], are marked by "\*", "+", "x" and "#" respectively. The other data points are from [6,7]. The error bars show combined statistical and systematic uncertainties. Three data points marked by "o" and the solid curve are the same as in Fig. 1a, and show the structure function at  $Q^2=230 \; (\text{GeV/c})^2$ . (b). The same data plotted as a function of the scaling variable  $\bar{x}$ .
- FIG. 4. Neutron-to-proton structure function ratio at large x. The solid line is the result of our analysis. The dotted lines show combined statistical and systematic errors. The dashed line is the quark model prediction for  $x \to 1$  for equal distributions of valence quarks. The dot-dashed line shows the expected behavior of this ratio from polynomial extrapolation of BCDMS and NMC data [4].

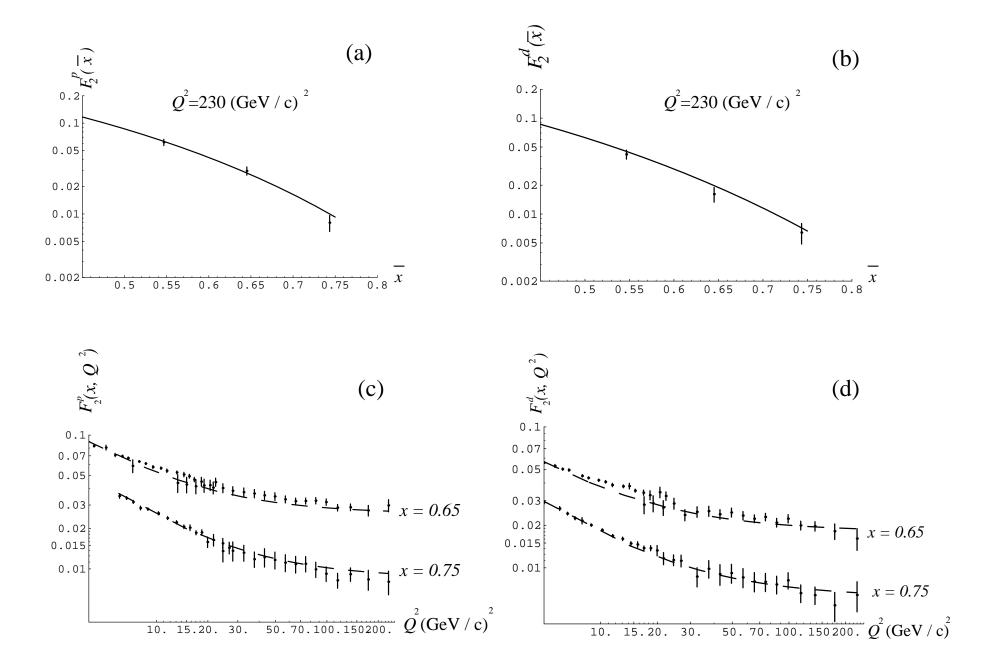


Fig. 1

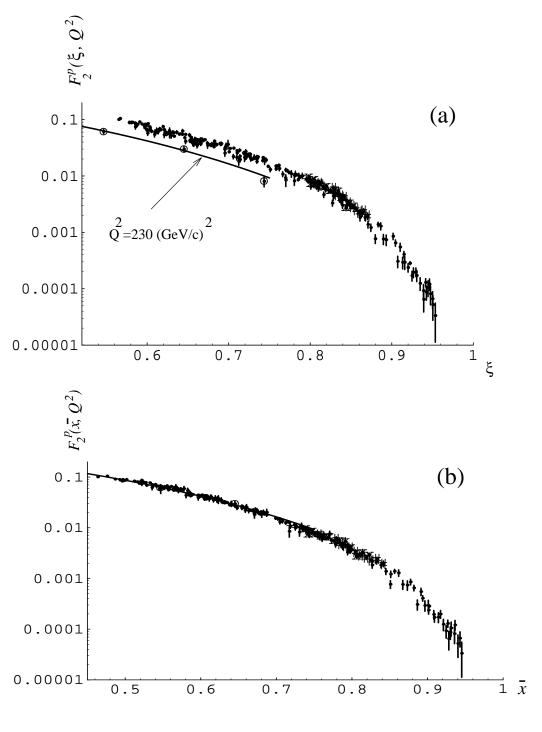


Fig. 2

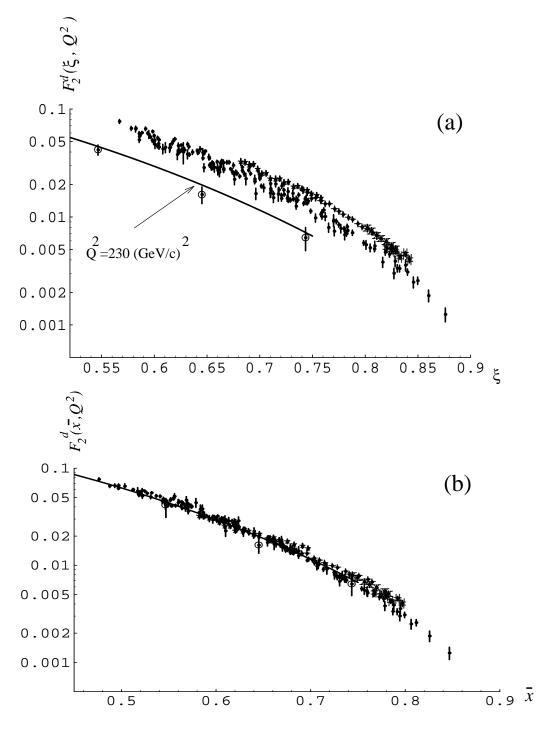


Fig. 3

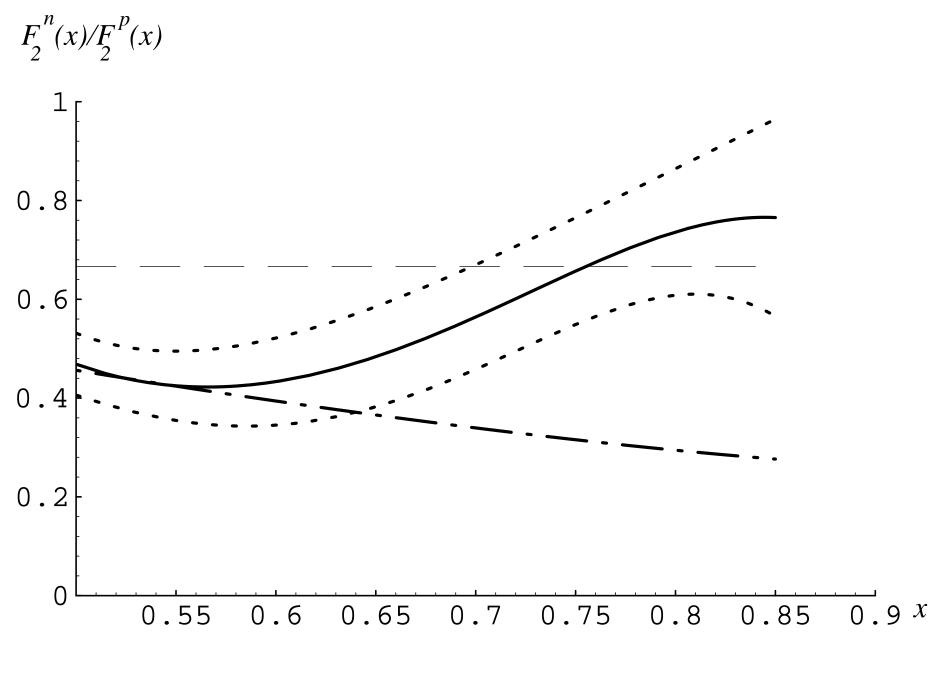


Fig.4